

Aspects of thick brane worlds: 4D gravity localization, smoothness, and mass gap

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Abstract

We review some interrelated aspects of thick brane worlds constructed within the framework of 5D gravity coupled to a scalar field depending on the extra dimension. It turns out that when analyzing localization of 4D gravity in this smooth version of the Randall-Sundrum model, a kind of dichotomy emerges. In the first case the geometry is completely smooth and the spectrum of metric fluctuations shows a single massless bound state, corresponding to the 4D graviton, and a tower of massive states described by a continuous spectrum of Kaluza-Klein excitations starting from zero mass, indicating the lack of a mass gap. In the second case, there are two bound states, a massless 4D graviton and a massive excitation, separated by a mass gap from a continuous spectrum of massive modes; nevertheless, the presence of a mass gap in the graviton spectrum of the theory is responsible for a naked singularity at the boundaries (or spatial infinity) of the Riemannian manifold. However, the imposition of unitary boundary conditions, which is equivalent to eliminating the continuous spectrum of gravitational massive modes, renders these singularities harmless from the physical point of view, providing the viability of the model.

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1 Introduction

In recent years, several results have been obtained in the framework of thick braneworld scenarios with a single extra dimension [1]–[11]. These models represent more realistic generalizations of several pioneer thin brane configurations [12]–[15], since if there are extra dimensions, they must be accessible at least at some hypothetical energy scale.

One way to smooth out the singular brane configurations is to replace the delta functions in the action of the system by a self-interacting scalar field. Once we have a smooth brane configuration, it is important from the phenomenological point of view to look for configurations that allow for the existence of a mass gap in the spectrum of gravitational Kaluza–Klein (KK) excitations. This gap fixes the energy scale at which massive modes can be excited and it is relevant for experimentally distinguishing the imprints of the massless zero mode, identified with a stable 4D graviton, from those coming from the massive modes, either discrete or continuous. When there is no mass gap in the spectrum, there are several modes with very small masses that cannot be experimentally distinguished from the massless one.

The relevance of braneworld models is related to the fact that we could live in a higher dimensional world without contradiction with up to day experimentally tested four-dimensional gravitational effects.

An interesting issue is related to the fact that naked singularities arise when a mass gap is present in the five-dimensional gravitational spectrum of the fluctuations of the of the system when studying four-dimensional gravity localization on thick brane configurations. In Sec. II it is shown that if one considers localization of 4D gravity with a generic warped ansatz for the metric, the presence of a mass gap in the spectrum of metric fluctuations necessarily implies the existence of naked singularities located at the boundaries (or spatial infinity, depending on the used coordinate system) of the fifth dimension or, alternatively, the five-dimensional manifold can be completely smooth iff the massive spectrum of KK excitations is gapless. This analysis is made in terms of a simple, but generic relationship existing between the 5D curvature scalar, the warp factor and the quantum mechanical potential which governs the dynamics of the linearized transverse traceless modes of metric fluctuations⁵. We follow a traditional point of view which states that spaces with naked singularities are physically acceptable only if one imposes unitary boundary conditions that guarantee four-dimensional energy and momentum conservation [2],[16],[17]. By analyzing the spectrum of KK modes when imposing unitary boundary conditions, one can see that the continuum part of the spectrum is projected out. Thus, the theory of the discrete part of the spectrum remains unitary because these modes die off rapidly enough as we approach the singularity, a fact that provides a viable model.

⁵It is important to stress that these computations are done for the sector in which the fluctuations of the scalar field vanish.

2 4D gravity localization: smoothness vs. mass gap

Let us consider the following five-dimensional action [1, 2]

$$S_5 = \int d^5x \sqrt{|G|} \left[-\frac{1}{4} R_5 + \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right], \quad (1)$$

where ϕ is a bulk scalar field and $V(\phi)$ is a self-interacting potential for the scalar field. We shall study a solution which preserves four-dimensional Poincaré invariance with the metric

$$ds_5^2 = e^{2A(y)} \eta_{mn} dx^m dx^n - dy^2, \quad (2)$$

where $e^{2A(y)}$ is the warp factor, $m, n = 0, 1, 2, 3$.

Let us study the metric fluctuations h_{mn} of (2) given by

$$ds_5^2 = e^{2A(y)} [\eta_{mn} + h_{mn}(x, y)] dx^m dx^n - dy^2. \quad (3)$$

In the general case one must consider the fluctuations of the scalar field when treating fluctuations of the classical background metric since they are coupled, however, following [1], we shall just study the transverse traceless modes of the background fluctuations h_{mn}^T since they decouple from the sector in which the scalar perturbations vanish.

In order to get a conformally flat metric we perform the coordinate transformation

$$dz = e^{-A} dy. \quad (4)$$

Thus, the equation that governs the dynamics of the transverse traceless modes of the metric fluctuations h_{mn}^T becomes [1, 2]

$$(\partial_z^2 + 3A'\partial_z - \square) h_{mn}^T(x, z) = 0, \quad (5)$$

where $A' = dA/dz$. This equation supports a massless and normalizable 4D graviton given by $h_{mn}^T = K_{mn} e^{ipx}$, where K_{mn} are constant parameters and $p^2 = m^2 = 0$.

We adopt the following ansatz

$$h_{mn}^T = e^{ipx} e^{-3A/2} \Psi_{mn}(z)$$

in order to recast (5) into a Schrödinger's equation form (here we have dropped the subscripts in Ψ for simplicity):

$$[\partial_z^2 - V_{QM}(z) + m^2] \Psi(z) = 0, \quad (6)$$

where m is the 4D mass of the KK excitation modes, and the analog quantum mechanical potential has the form

$$V_{QM}(z) = \frac{3}{2} \left[\partial_z^2 A + \frac{3}{2} (\partial_z A)^2 \right]. \quad (7)$$

As it is shown in [3], from (6) it follows that the zero energy wave function is given by

$$\Psi_0(z) = e^{\frac{3}{2}A(z)}. \quad (8)$$

The condition for localizing four-dimensional gravity demands $\Psi_0(z)$ to be normalizable; in other words, the zero energy wave function must satisfy the following relationship

$$\int dz |\Psi_0|^2 < \infty. \quad (9)$$

Finally, it is worth noticing that the curvature scalar corresponding to the ansatz (2) in the z coordinate adopts the form

$$R_5 = 8e^{-2A} \left[\partial_z^2 A + \frac{3}{2} (\partial_z A)^2 \right]. \quad (10)$$

As it was pointed out in [3, 18] in order to have a localized massless mode of the 5D graviton, V_{QM} must be a well potential with a negative minimum which approaches a positive value $V_{QM\infty} \geq 0$ as $|z| \rightarrow \infty$. This property ensures the fulfilment of the requirement (9).

Moreover, if we wish to have a well defined effective field theory, it is desirable to obtain solutions where the massless graviton is separated from the massive modes by a mass gap [18]. This phenomenological aspect ensures the lack of arbitrary light KK excitations in the mass spectrum. If the quantum mechanical potential asymptotically approaches a positive value $V_{QM\infty} > 0$, the existence of a mass gap is guaranteed.

Let us determine how these two aspects are related to the smooth character of the curvature scalar. Thus, we must study the relation among the following aspects of the theory:

- a) Localization of 4D gravity in the 5D braneworld.
- b) Smoothness of the curvature scalar R_5 .
- c) Existence of a mass gap in the spectrum of gravitational excitations of the system.

From the expressions for Ψ_0 (8), V_{QM} (7) and R_5 (10) we find that the curvature scalar can be written in terms of the zero energy wave function and the quantum mechanical potential as follows:

$$R_5 = \frac{16}{3} \Psi_0^{-\frac{4}{3}} V_{QM}. \quad (11)$$

Since the warp factor is real, (8) and (9) imply that Ψ_0 must tend to zero asymptotically

$$\Psi_0|_{z \rightarrow \infty} \rightarrow 0 \quad (12)$$

in order to ensure 4D gravity localization. If we indeed require the existence of a mass gap, $V_{QM\infty}$ must adopt a positive value asymptotically.

Recall that we are considering thick brane configurations that are regular at the position of the brane. Thus, if a) and c) are fulfilled, (11) implies that R_5 will necessary possess

naked singularities as $|z| \rightarrow \infty$. On the other side, if we assume that conditions a) and b) are satisfied, then V_{QM} must vanish asymptotically at least with the same rapidity of $\Psi_0^{-\frac{4}{3}}$, implying, in turn, that there is no mass gap for such a solution of the model.

Thus, if one desires to construct a completely regular thick brane configuration in the framework of the model (1) with the ansatz (2) in which 4D gravity is localized, then the corresponding mass spectrum will have no mass gap between the zero mode bound state, identified with a stable 4D graviton, and the continuous spectrum of massive KK excitations.

Conversely, if we require the existence of a mass gap in such a model restricted by (2), then the scalar curvature will necessary develop naked singularities asymptotically. This singularities can be made harmless if one imposes unitary boundary conditions in the spirit of [2],[16],[17], providing a viable model from the physical point of view.

The fact that the manifold develops naked singularities at the boundaries in this case does not matter if no conserved quantities are allowed to leak out through the boundaries. The Poincaré isometries of metric (2) corresponds to 4D conservation laws. Thus, by considering the translations generated by the Killing vectors $\xi_m^\mu = \delta_m^\mu$, where μ is a five-dimensional index, one can construct currents by contracting with the stress tensor

$$J^\mu = T^{\mu\nu} \xi_\nu^m. \quad (13)$$

These currents satisfy a covariant conservation law of 4D energy and momentum

$$\frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} J^\mu) = 0. \quad (14)$$

Therefore we demand that the flux through the singular boundary of spacetime (along the transverse direction) must vanish for all currents in order to ensure that these quantities are conserved in the presence of a singularity:

$$\lim_{z \rightarrow \infty} \sqrt{g} J^z = \lim_{z \rightarrow \infty} \sqrt{g} g^{zz} \frac{1}{2} \partial_m h_{pq}^T \partial_z h_{pq}^T = 0. \quad (15)$$

We shall see below that this unitary boundary conditions project out all continuum modes from the spectrum of KK graviton excitations, leading to a unitary spectrum consisting of discrete bound states in the theory.

2.1 Solution with a mass gap

The system under consideration allows for the following solution [1, 2]

$$\begin{aligned} e^{2A} &= [\cos(ay - y_0)]^b, & \phi &= \frac{\sqrt{3b}}{2} \ln [\sec(ay - y_0) + \tan(ay - y_0)], \\ V(\phi) &= \frac{3a^2 b^2}{4} \left[1 + \frac{1-2b}{b} \cosh^2 \left(\frac{2\phi}{\sqrt{3b}} \right) \right]. \end{aligned} \quad (16)$$

where a, b and y_0 are arbitrary constants.

Such a solution describes a thick brane located at y_0 when the range of the fifth dimension is $-\pi/2 \leq a(y - y_0) \leq \pi/2$ [8] and involves naked singularities at the boundaries of the manifold [2, 9]; a represents the inverse of the brane's width ($\Delta \sim 1/a$) when $b > 0$.

If $b = 2$ one can invert the coordinate transformation (4) yielding the following relation:

$$\cos[a(y - y_0)] = \text{sech}(az).$$

This transformation decompactifies the fifth dimension and sends the naked singularities that were present at $-\pi/2a$ and $\pi/2a$ to spatial infinity [2, 9]. In terms of z , the warp factor becomes

$$A(z) = \ln \text{sech}(az),$$

and the analog quantum mechanical potential adopts the form [9]

$$V_{QM}(z) = \frac{3a^2}{4} \{3 - 5\text{sech}^2(az)\}. \quad (17)$$

Recall that the eigenvalue spectrum denoted by m^2 parameterizes the spectrum of graviton masses that a four-dimensional observer standing at $z = 0$ sees. The analog quantum mechanical potential asymptotically approaches a positive value $V_{QM}(\infty) = 9a^2/4$, leading to a mass gap between the zero mode bound state and the first KK massive mode [3]. By setting $u = az$ the Schrödinger equation can be transformed into the standard form possessing a modified Pöschl–Teller potential with $n = 3/2$ (see [19],[9])

$$\left[-\partial_u^2 - n(n+1)\text{sech}^2 u\right] \Psi(u) = \left[\frac{m^2}{a^2} - \frac{9}{4}\right] \Psi(u) = E\Psi(u). \quad (18)$$

Thus, the wave function possesses two bound states: the ground state Ψ_0 with energy E_0 and an excited massive state Ψ_1 with energy E_1 . In this case the Schrödinger equation (6) can be solved analytically for arbitrary m and the general solution is a linear combination of associated Legendre functions of first and second kind of degree $3/2$ and order $\mu = \sqrt{\frac{9}{4} - \frac{m^2}{a^2}}$:

$$\Psi_m = k_1 P_{3/2}^\mu[\tanh(az)] + k_2 Q_{3/2}^\mu[\tanh(az)], \quad (19)$$

where k_1 and k_2 are integration constants. The ground state corresponds to the massless state $m = 0$ ($\mu = 3/2$) and possesses energy $E_0 = -n^2 = -9/4$, whereas the excited state has mass $m = \sqrt{2}a$ ($\mu = 1/2$) and energy $E_1 = -(n-1)^2 = -1/4$.

Since for the special values $\mu = 3/2$ and $\mu = 1/2$ the associated Legendre function's series are finite, one obtains the following eigenfunction for the massless mode

$$\Psi_0 = C_0 \text{sech}^{3/2}(az), \quad (20)$$

whereas the excited massive mode adopts the form

$$\Psi_1 = C_1 \sinh(az)\text{sech}^{3/2}(az), \quad (21)$$

where C_0 and C_1 are arbitrary normalization constants. Since (20) represents the lowest energy eigenfunction of the Schrödinger equation (6) and it has no zeros, it can be interpreted as a massless 4D graviton with no tachyonic instabilities from modes with $m^2 < 0$. On the other side, the eigenfunction (21) represents a normalizable massive graviton localized on the center of the thick brane. The mass gap existing between these two bound states eliminates the potentially dangerous arbitrarily light KK excitations, sending them to energies of the scale of a .

In the spectrum there is also a continuous part of massive modes with $2m > 3a$ (the order becomes purely imaginary $\mu = i\rho$, where $\rho = \sqrt{(m/a)^2 - 9/4}$) that describe plane waves as they approach spatial infinity [9] and give rise to small corrections to Newton's law in 4D flat spacetime in the thin brane limit [3, 10].

However, by imposing the unitary boundary conditions mentioned above, i.e., by demanding the vanishing of the flux through the singular boundary of our spacetime:

$$\lim_{z \rightarrow \infty} e^{3A/2} \Psi(z) \partial_z \left[e^{-3A/2} \Psi(z) \right] \sim \Psi(z) [(3B - 2A\rho) \cos(a\rho z) + (3A + 2B\rho) \sin(a\rho z)] = 0, \quad (22)$$

where we have made use of the asymptotic form of $\Psi(z)$:

$$\Psi(z) \sim A \sin(a\rho z) + B \cos(a\rho z), \quad (23)$$

we get a result implying that A and B must vanish, just eliminating all the continuum modes from the spectrum of KK excitations. Thus, the unitary spectrum consists of two discrete modes because they do not generate any flux into the singularity.

3 Conclusions and discussion

We have a 5D thick braneworld arising in a scalar tensor theory and have investigated the relationship existing among the localization of 4D gravity, the smooth or singular character of the curvature scalar of the classical background, and the existence of a mass gap which separates the massless graviton bound state from the massive KK excitations. We have shown that when considering 4D gravity localization in the 5D thick braneworld, the smooth character of the scalar curvature of the manifold excludes the possibility of the existence of a mass gap in the graviton spectrum of fluctuations; conversely, by demanding the existence of a mass gap between the massless and massive modes of KK excitations, the curvature scalar necessarily develops naked singularities at the boundaries of the manifold. We further considered an explicit solution which gives rise to a mass gap in the spectrum of graviton fluctuations. This solution contains a massless graviton, one massive excited state with $m = \sqrt{2}a$, and a continuum of modes with a mass gap of size $m = 3a/2$. At very low energies, none of these massive modes can be excited, and an observer at $z = 0$ sees pure four-dimensional gravity. At higher energies the massive state can be excited, giving some corrections to Newton's law and, finally, at energies larger than the gap the whole continuum of modes can be excited. Violations of unitarity occur only when modes

that can travel out to the singularities can be excited, i.e. only at energies above the mass of the lightest continuum mode.

For this reason we imposed unitary boundary conditions on the described solution, a fact that eliminates all the continuous modes from the spectrum of KK excitations of the classical background and renders the naked singularities harmless (in the same spirit as in [2],[16],[17]).

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Abstract

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One way to smooth out the singular brane configurations is to replace the delta functions in the action of the system by a self-interacting scalar field. Once we have a smooth brane configuration, it is important from the phenomenological point of view to look for configurations that allow for the existence of a mass gap in the spectrum of gravitational Kaluza–Klein (KK) excitations. This gap fixes the energy scale at which massive modes can be excited and it is relevant for experimentally distinguishing the imprints of the massless zero mode, identified with a stable 4D graviton, from those coming from the massive modes, either discrete or continuous. When there is no mass gap in the spectrum, there are several modes with very small masses that cannot be experimentally distinguished from the massless one.

The relevance of braneworld models is related to the fact that we could live in a higher dimensional world without contradiction with up to day experimentally tested 4D gravitational effects.

An interesting issue is related to the fact that naked singularities arise when a mass gap is present in the 5D spectrum of transverse traceless modes of metric fluctuations when studying 4D gravity localization on thick brane configurations. In Sec. II it is shown that if one considers localization of 4D gravity with a generic warped ansatz for the metric, the presence of a mass gap in the spectrum of the transverse traceless fluctuations necessarily implies the existence of naked singularities located at the boundaries (or spatial infinity, depending on the used coordinate system) of the fifth dimension or, alternatively, the 5D manifold can be completely smooth iff the massive spectrum of KK excitations is gapless. This analysis is made in terms of a simple, but generic relationship existing between the 5D curvature scalar, the warp factor and the quantum mechanical potential which governs the dynamics of the linearized transverse traceless modes of metric fluctuations. We follow a traditional point of view which states that spaces with naked singularities are physically acceptable only if one imposes unitary boundary conditions that guarantee 4D energy and momentum conservation [2],[53],[54]. By analyzing the spectrum of KK modes when imposing unitary boundary conditions, one can see that the continuum part of the spectrum is projected out. Thus, the theory of the discrete part of the spectrum remains unitary because these modes die off rapidly enough as we approach the singularity, a fact that provides a viable model.

2 4D gravity localization: smoothness vs. mass gap

Let us consider the following 5D action [1, 2]

$$S_5 = \int d^5x \sqrt{|G|} \left[-\frac{1}{4} R_5 + \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right], \quad (1)$$

where ϕ is a bulk scalar field and $V(\phi)$ is a self-interacting potential for the scalar field. We shall study a solution which preserves 4D Poincaré invariance with the metric

$$ds_5^2 = e^{2A(y)} \eta_{mn} dx^m dx^n - dy^2, \quad (2)$$

where $e^{2A(y)}$ is the warp factor, $m, n = 0, 1, 2, 3$.

In the general case the analysis of classical gravitational perturbations in braneworld models may be complicated because the metric fluctuations are coupled to the fluctuations of the scalar field. On the other hand, the invariance of the theory under diffeomorphisms makes the very definition of perturbations gauge dependent, in other words, by performing a small-amplitude transformation of the space-time coordinates (a gauge transformation), we can easily introduce “fictitious” perturbations in the background space-time. There are two approaches to deal with these gauge ambiguities. The first one is to fix a gauge, i. e. to pick a coordinate system which completely eliminate the gauge freedom; the second one consists of extracting some gauge-invariant variables to obtain coordinate independent results [55]. In our work we consider the gauge-invariant method.

If 4D Poincaré invariance is unbroken, the different modes of the geometry should be classified according to 4D Poincaré transformations in scalar, vector and tensor sectors [55], where the vector modes are divergenceless and the tensor modes are transverse and traceless.

The scalar and vector fluctuations of the geometry are not invariant under a general 5D infinitesimal coordinate transformation (i. e. gauge-invariant), so that suitable gauge invariant variables can be chosen in order to study the perturbation equations in a gauge-invariant framework [55]. On the other hand, the tensor modes are automatically invariant under a general 5D infinitesimal coordinate transformation. Furthermore, in linear theory of perturbations, there is no coupling between the different fluctuation modes, and hence they evolve independently. Following [1], we shall just study the transverse traceless modes of the background fluctuations h_{mn}^T since they decouple from the sector of the scalar field perturbations.

Let us study the metric fluctuations h_{mn}^T of (2) given by

$$ds_5^2 = e^{2A(y)} \left[\eta_{mn} + h_{mn}^T(x, y) \right] dx^m dx^n - dy^2, \quad (3)$$

where $\eta^{mn} h_{mn}^T = \partial^m h_{mn}^T = 0$.

In order to get a conformally flat metric we perform the coordinate transformation

$$dz = e^{-A} dy. \quad (4)$$

Thus, the equation that governs the dynamics of the transverse traceless modes of the metric fluctuations h_{mn}^T becomes [1, 2]

$$\left(\partial_z^2 + 3A'\partial_z - \square\right) h_{mn}^T(x, z) = 0, \quad (5)$$

where $A' = dA/dz$. This equation supports a massless and normalizable 4D graviton given by $h_{mn}^T = K_{mn}e^{ipx}$, where K_{mn} are constant parameters and $p^2 = m^2 = 0$.

We adopt the following ansatz

$$h_{mn}^T = e^{ipx} e^{-3A/2} \Psi_{mn}(z)$$

in order to recast (5) into a Schrödinger's equation form (here we have dropped the subscripts in Ψ for simplicity):

$$[\partial_z^2 - V_{QM}(z) + m^2]\Psi(z) = 0, \quad (6)$$

where m is the 4D mass of the KK excitation modes, and the analog quantum mechanical potential has the form

$$V_{QM}(z) = \frac{3}{2} \left[\partial_z^2 A + \frac{3}{2} (\partial_z A)^2 \right]. \quad (7)$$

As it is shown in [4], from (6) it follows that the zero energy wave function is given by

$$\Psi_0(z) = e^{\frac{3}{2}A(z)}. \quad (8)$$

The condition for localizing 4D gravity demands $\Psi_0(z)$ to be normalizable; in other words, the zero energy wave function must satisfy the following relationship

$$\int dz |\Psi_0|^2 < \infty. \quad (9)$$

Finally, it is worth noticing that the curvature scalar corresponding to the ansatz (2) in the z coordinate adopts the form

$$R_5 = 8e^{-2A} \left[\partial_z^2 A + \frac{3}{2} (\partial_z A)^2 \right]. \quad (10)$$

As it was pointed out in [4, 56] in order to have a localized massless mode of the 5D graviton, V_{QM} must be a well potential with a negative minimum which approaches a positive value $V_{QM\infty} \geq 0$ as $|z| \rightarrow \infty$. This property ensures the fulfilment of the requirement (9).

Moreover, if we wish to have a well defined effective field theory, it is desirable to obtain solutions where the massless graviton is separated from the massive modes by a mass gap [56]. This phenomenological aspect ensures the lack of arbitrary light KK excitations in the mass spectrum. If the quantum mechanical potential asymptotically approaches a positive value $V_{QM\infty} > 0$, the existence of a mass gap is guaranteed.

Let us determine how these two aspects are related to the smooth character of the curvature scalar. Thus, we must study the relation among the following aspects of the theory:

- a) Localization of 4D gravity in the 5D braneworld.
- b) Smoothness of the curvature scalar R_5 .
- c) Existence of a mass gap in the spectrum of gravitational excitations of the system.

From the expressions for Ψ_0 (8), V_{QM} (7) and R_5 (10) we find that the curvature scalar can be written in terms of the zero energy wave function and the quantum mechanical potential as follows:

$$R_5 = \frac{16}{3} \Psi_0^{-\frac{4}{3}} V_{QM}. \quad (11)$$

Since the warp factor is real, (8) and (9) imply that Ψ_0 must tend to zero asymptotically

$$\Psi_0|_{z \rightarrow \infty} \rightarrow 0 \quad (12)$$

in order to ensure 4D gravity localization. If we indeed require the existence of a mass gap, $V_{QM\infty}$ must adopt a positive value asymptotically.

Recall that we are considering thick brane configurations that are regular at the position of the brane. Thus, if a) and c) are fulfilled, (11) implies that R_5 will necessarily possess naked singularities as $|z| \rightarrow \infty$. On the other side, if we assume that conditions a) and b) are satisfied, then V_{QM} must vanish asymptotically at least with the same rapidity of $\Psi_0^{-\frac{4}{3}}$, implying, in turn, that there is no mass gap for such a solution of the model.

Thus, if one desires to construct a completely regular thick brane configuration in the framework of the model (1) with the ansatz (2) in which 4D gravity is localized, then the corresponding mass spectrum will have no mass gap between the zero mode bound state, identified with a stable 4D graviton, and the continuous spectrum of massive KK excitations.

Conversely, if we require the existence of a mass gap in such a model restricted by (2), then the scalar curvature will necessarily develop naked singularities asymptotically. This singularities can be made harmless if one imposes unitary boundary conditions in the spirit of [2],[53] and [54], providing a viable model from the physical point of view.

The fact that the manifold develops naked singularities at the boundaries in this case does not matter if no conserved quantities are allowed to leak out through the boundaries. The 4D Poincaré isometries of metric (2) corresponds to 4D conservation laws. Thus, by considering the translations generated by the Killing vectors $\xi_m^\mu = \delta_m^\mu$, where μ is a 5D index, one can construct currents by contracting with the stress tensor [2]

$$J^\mu = T^{\mu\nu} \xi_\nu^m. \quad (13)$$

These currents satisfy a covariant conservation law of 4D energy and momentum

$$\frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} J^\mu) = 0. \quad (14)$$

Therefore we demand that the flux through the singular boundary of spacetime (along the transverse direction) must vanish for all currents in order to ensure that these quantities are conserved in the presence of a singularity:

$$\lim_{z \rightarrow \infty} \sqrt{g} J^z = \lim_{z \rightarrow \infty} \sqrt{g} g^{zz} \frac{1}{2} \partial_m h_{pq}^T \partial_z h_{pq}^T = 0. \quad (15)$$

We shall see below that this unitary boundary conditions project out all continuum modes from the spectrum of KK graviton excitations, leading to a unitary spectrum consisting of discrete bound states in the theory.

2.1 Solution with a mass gap

The system under consideration allows for the following solution [1, 2]

$$e^{2A} = [\cos(ay - y_0)]^b, \quad \phi = \frac{\sqrt{3b}}{2} \ln [\sec(ay - y_0) + \tan(ay - y_0)], \quad (16)$$

$$V(\phi) = \frac{3a^2 b^2}{4} \left[1 + \frac{1 - 2b}{b} \cosh^2 \left(\frac{2\phi}{\sqrt{3b}} \right) \right].$$

where a, b and y_0 are arbitrary constants.

Such a solution describes a thick brane located at y_0 when the range of the fifth dimension is $-\pi/2 \leq a(y - y_0) \leq \pi/2$ (see [28]) and involves naked singularities at the boundaries of the manifold [2, 30]; a represents the inverse of the brane's width ($\Delta \sim 1/a$) when $b > 0$.

If $b = 2$ one can invert the coordinate transformation (4) yielding the following relation:

$$\cos[a(y - y_0)] = \text{sech}(az).$$

This transformation decompactifies the fifth dimension and sends the naked singularities that were present at $-\pi/2a$ and $\pi/2a$ to spatial infinity [2, 30]. In terms of z , the warp factor becomes

$$A(z) = \ln \text{sech}(az),$$

and the analog quantum mechanical potential adopts the form [30]

$$V_{QM}(z) = \frac{3a^2}{4} \{3 - 5 \text{sech}^2(az)\}. \quad (17)$$

Recall that the eigenvalue spectrum denoted by m^2 parameterizes the spectrum of graviton masses that a 4D observer standing at $z = 0$ sees. The analog quantum mechanical potential asymptotically approaches a positive value $V_{QM}(\infty) = 9a^2/4$, leading to a mass gap between the zero mode bound state and the first KK massive mode [4]. By setting $u = az$ the Schrödinger equation can be transformed into the standard form possessing a modified Pöschl–Teller potential with $n = 3/2$ (see [30],[57])

$$\left[-\partial_u^2 - n(n+1) \text{sech}^2 u \right] \Psi(u) = \left[\frac{m^2}{a^2} - \frac{9}{4} \right] \Psi(u) = E \Psi(u). \quad (18)$$

Thus, the wave function possesses two bound states: the ground state Ψ_0 with energy E_0 and an excited massive state Ψ_1 with energy E_1 . In this case the Schrödinger equation (6) can be solved analytically for arbitrary m and the general solution is a linear combination of associated Legendre functions of first and second kind of degree $3/2$ and order $\mu = \sqrt{\frac{9}{4} - \frac{m^2}{a^2}}$:

$$\Psi_m = k_1 P_{3/2}^\mu [\tanh(az)] + k_2 Q_{3/2}^\mu [\tanh(az)], \quad (19)$$

where k_1 and k_2 are integration constants. The ground state corresponds to the massless state $m = 0$ ($\mu = 3/2$) and possesses energy $E_0 = -n^2 = -9/4$, whereas the excited state has mass $m = \sqrt{2}a$ ($\mu = 1/2$) and energy $E_1 = -(n-1)^2 = -1/4$.

Since for the special values $\mu = 3/2$ and $\mu = 1/2$ the associated Legendre function's series are finite, one obtains the following eigenfunction for the massless mode

$$\Psi_0 = C_0 \operatorname{sech}^{3/2}(az), \quad (20)$$

whereas the excited massive mode adopts the form

$$\Psi_1 = C_1 \sinh(az) \operatorname{sech}^{3/2}(az), \quad (21)$$

where C_0 and C_1 are arbitrary normalization constants. Since (20) represents the lowest energy eigenfunction of the Schrödinger equation (6) and it has no zeros, it can be interpreted as a massless 4D graviton with no tachyonic instabilities from modes with $m^2 < 0$. On the other side, the eigenfunction (21) represents a normalizable massive graviton localized on the center of the thick brane. The mass gap existing between these two bound states eliminates the potentially dangerous arbitrarily light KK excitations, sending them to energies of the scale of a .

In the spectrum there is also a continuous part of massive modes with $2m > 3a$ (the order becomes purely imaginary $\mu = i\rho$, where $\rho = \sqrt{(m/a)^2 - 9/4}$) that describe plane waves as they approach spatial infinity [30] and give rise to small corrections to Newton's law in 4D flat spacetime in the thin brane limit [4, 31].

However, by imposing the unitary boundary conditions mentioned above, i.e., by demanding the vanishing of the flux through the singular boundary of our spacetime:

$$\lim_{z \rightarrow \infty} e^{3A/2} \Psi(z) \partial_z \left[e^{-3A/2} \Psi(z) \right] \sim \Psi(z) [(3B - 2A\rho) \cos(a\rho z) + (3A + 2B\rho) \sin(a\rho z)] = 0, \quad (22)$$

where we have made use of the asymptotic form of $\Psi(z)$:

$$\Psi(z) \sim A \sin(a\rho z) + B \cos(a\rho z), \quad (23)$$

we get a result implying that A and B must vanish, just eliminating all the continuum modes from the spectrum of KK excitations. Thus, the unitary spectrum consists of two discrete modes because they do not generate any flux into the singularity.

3 Conclusions and discussion

We have a 5D thick braneworld arising in a scalar tensor theory and have investigated the relationship existing among the localization of 4D gravity, the smooth or singular character of the curvature scalar of the classical background, and the existence of a mass gap which separates the massless graviton bound state from the massive KK excitations. We have shown that when considering 4D gravity localization in the 5D thick braneworld, the smooth character of the scalar curvature of the manifold excludes the possibility of the existence of a mass gap in the graviton spectrum of fluctuations; conversely, by demanding the existence of a mass gap between the massless and massive modes of KK excitations, the curvature scalar necessarily develops naked singularities at the boundaries of the manifold. We further considered an explicit solution which gives rise to a mass gap in the spectrum of graviton fluctuations. This solution contains a massless graviton, one massive excited state with $m = \sqrt{2}a$, and a continuum of modes with a mass gap of size $m = 3a/2$. At very low energies, none of these massive modes can be excited, and an observer at $z = 0$ sees pure 4D gravity. At higher energies the massive state can be excited, giving some corrections to Newton's law and, finally, at energies larger than the gap the whole continuum of modes can be excited. Violations of unitarity occur only when modes that can travel out to the singularities can be excited, i.e. only at energies above the mass of the lightest continuum mode.

For this reason we imposed unitary boundary conditions on the described solution, a fact that eliminates all the continuous modes from the spectrum of KK excitations of the classical background and renders the naked singularities harmless (in the same spirit as in [2],[53],[54]).

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